

# MATH 121A Prep: Proofs

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## Facts to Know:

Induction: Proof technique for showing a statement is true for all for all positive integers

- Base Case:
- Inductive Step:

Quantifiers:

- There exists:
- For all:

Negation of quantifiers:

- Negation of there exists:
- Negation of for all:

## Examples:

1. Let  $A = \begin{bmatrix} a & b-a \\ 0 & b \end{bmatrix}$ . Prove the formula  $A^n = \begin{bmatrix} a^n & b^n - a^n \\ 0 & b^n \end{bmatrix}$

2. Convert the following statements between words and mathematical notation:

(a) For all objects  $y$  in a set  $Y$  there exists an  $x$  in  $X$  such that  $f(x) = y$ .

(b)  $\exists \vec{v}_1, \vec{v}_2 \in \mathbb{R}^2$  such that  $\forall \vec{w} \in \mathbb{R}^2 \exists! c_1, c_2$  where  $\vec{w} = c_1 \vec{v}_1 + c_2 \vec{v}_2$

3. Write the negation of the above statements: